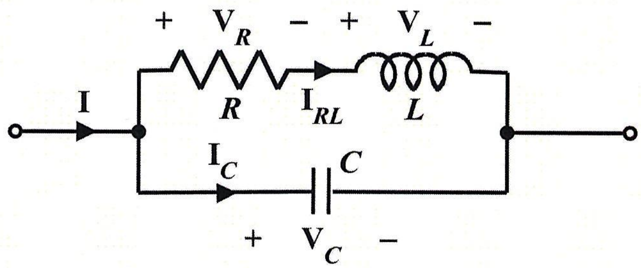
EE 3340

Homework Problem #019

The circuit shown below is sometimes called a "tuning circuit."



If $R = 20\Omega$, $L = 5\,\text{mH}$, and $C = 0.5\,\mu\text{F}$:

a. Determine the frequency, ω, at which its equivalent impedance, $\mathbf{Z}_{eq} = \frac{\mathbf{V}_C}{\mathbf{I}}$, is purely resistive.

$$Z_{eq} = \frac{1}{j\omega C} \,\|\, (R + j\omega L) = \frac{\frac{1}{j\omega C}(R + j\omega L)}{\frac{1}{j\omega C} + R + j\omega L} = \frac{\frac{L}{C} - j\frac{R}{\omega C}}{R + j(\omega L - \frac{1}{\omega C})} \cdot \frac{R - j(\omega L - \frac{1}{\omega C})}{R - j(\omega L - \frac{1}{\omega C})}$$

$$= \frac{\frac{RL}{C} - \frac{R}{\omega C}(\omega L - \frac{1}{\omega C}) - j\frac{R^2}{\omega C} - j\frac{L}{C}(\omega L - \frac{1}{\omega C})}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$\text{Im}\{Z_{eq}\} = 0 \Rightarrow \frac{R^2}{\omega C} + \frac{L}{C}(\omega L - \frac{1}{\omega C}) = 0 \Rightarrow \left(\frac{L}{C} - R^2\right)\frac{1}{\omega} = L^2\omega$$

$$\text{or} \quad \omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2} \Rightarrow \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

∴ With the given component values, $\omega = 19.6\,\text{k}\,\frac{\text{rad}}{\text{s}}$

b. For the frequency determined in part a, determine that equivalent resistance.

$$\text{Re}\{Z_{eq}\} = \frac{\frac{RL}{C} - \frac{R}{\omega C}(\omega L - \frac{1}{\omega C})}{R^2 + (\omega L - \frac{2}{\omega C})^2} = 500\,\Omega$$